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ABSTRACT

This collection includes 50 mathematical games and puzzles for classroom use at all grade levels. Also included is a wide variety of activities with cubes, flash cards, graphs, dots, number patterns, geometric shapes, cross-number puzzles, and magic squares. (MM)

THE MICHIGAN COUNCIL OF TEACHERS OF MATHEMATICS

Guidelines for Quality Mathematics Programs

A Monograph Series

M C T M

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Foreward

This monograph is the first in a series to be published by the Guidelines Committee of the Michigan Council of Teachers of Mathematics. This committee is jointly funded by the MCTM and the Michigan Education Association.

During the first year the committee has held a hearing at each mathematics conference in the state and these hearings have been used to establish priorities of topics for the monographs. The contents have been contributed by teachers and students throughout the state. Our thanks go to scores of teachers willing to share their successful motivational activities with all MCTM members in Michigan. We would particularly like to thank Mr. William Kumbier and the teachers and students of Livonia Public schools for their contributions. Other sources for similar materials are: The Arithmetic Teacher, The Mathematics Teacher, Instructor, and The Grade Teacher.

The contents are organized by type rather than by artificial grade levels. Mathematical games and puzzles are for everyone. We hope that, through the use of these suggestions, your students of all ages will have fun and, in having fun, gain new insights and appreciations for the greatest of all the academic disciplines--------MATHEMATICS.

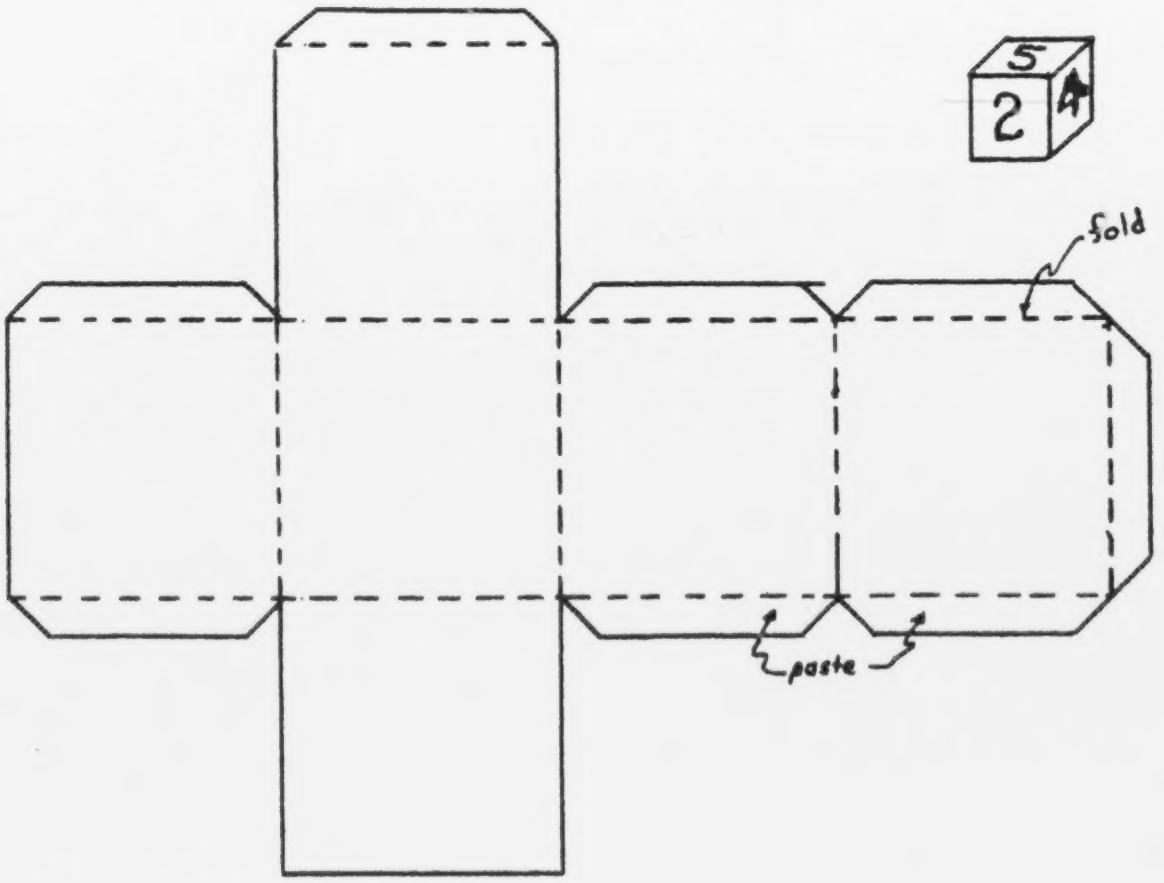
Terry Denman
Bea Munro
Roy Thompson

TABLE OF CONTENTS

Type of Activity	Pages
Cubes	3-5
Cards	6-12
Graphs, Lines Patterns, Dots	13-23
Crossword (number) puzzles	24-26
Puzzles, Codes, Magic Squares	27-39
Criss-cross, Tic-Tac-Toe	40-43
Games	44-52

Sumo

Large cardboard dice help in learning addition combinations. From oak tag each child made two blocks each two inches square using the pattern below. A number was written on each square with crayon before folding. The blocks were closed with tape. Each child tossed the two blocks and added the numbers that turned up. This is a good drill for children to do in pairs.



Twenty-One

I made cubes (cut off and sanded lengths of 1 x 1) and put on numbers as a dice, 1 opposite 6, 2 opposite 5, 3 opposite 4. The first player rolls the cube, the second can roll the cube on any of the four edges and add this number to the first number. Rules are the same as above. The advantage here is that the kids have something concrete to play with and seem to love it. If 21 becomes too simple, increase to 31 or 41.

Games Played with Number Cubes, Rolled Like Dice

Each player takes four cubes, three with numerals 0 through 9 and one with the four operation signs (+, -, x, ÷).

To begin the play, one player rolls the single "goal" cube, which is a different color and is marked with 2, 3, 4, 5, 6, 7, (or 9). The number that comes up becomes the goal for the round.

The players then roll their own sets of four cubes, all at the same time. Each player tries to arrange his cubes so that the resulting number (sum, difference, product or quotient) is a multiple of the number on the goal cube.

Examples:

The goal is [3]

You roll [3] [7] [2] [+]

Possible arrangements are:

27 + 3 (30)
32 + 7 (39)
72 + 3 (75)

You roll [2] [7] [5] [x]

Possible arrangements are:

27 x 5 (135)
57 x 2 (114)

You roll [6] [8] [2] [+]

No successful arrangement is possible, so you get a zero for the round.

You roll [8] [1] [4] [-]

Possible arrangements are:

41 - 8 (33)
14 - 8 (6)

Each player who is successful scores one point and the points are recorded on a score sheet.

The play proceeds to another round, with the next player on the left rolling the goal cube.

The first player to get ten points wins the game.

NOTE: A game set for four players can be made with seventeen one-inch cubes of three different colors marked with a black magic marker. Or plain cubes can be used, but marked with different colors to distinguish the goal cube, the numeral cubes, and the operation cubes.

Flip-It

This game is played with two cubes marked with the numerals 1, 2, 3, 4, 5, 6, or with two ordinary dice.

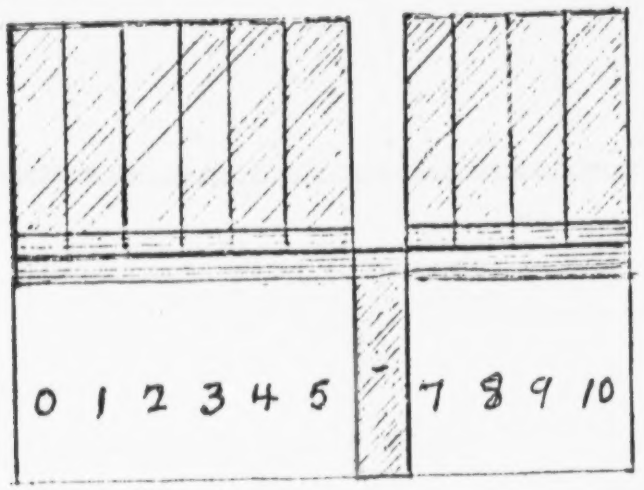
Each player has a Flip-It card which has numbers printed on it. Attached above each number is a cardboard strip which may be flipped down to cover it. The numbers are uncovered at the beginning of play.

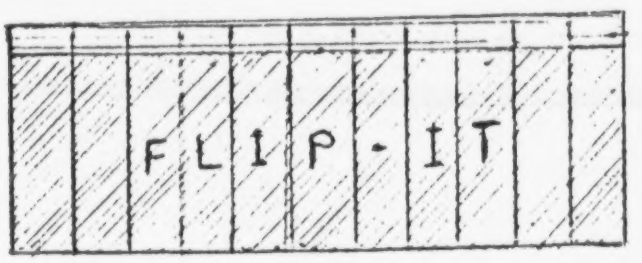
The first player rolls his cubes. He then decides what operation he will use so that the result will be a number he needs. With a 2 and a 4, he may add, subtract, multiply or divide. If he chooses to add, he makes a 6 and flips the cardboard strip down over the 6. If he chooses to subtract, he covers the 2; or, if he multiplies, he covers the 8. If a pair of numbers comes up that cannot be used because the resulting numbers are already covered, the player loses his turn.

The players proceed in order. The first player to cover all the numbers on his card calls "Flip-It" and wins the game.

Flip-it games can be made of posterboard. Take two contrasting colored pieces each four inches by eleven inches and fasten them together on the long sides to make a folding card. Make a binding of mystic tape or of contact paper cut into one and a fourth inch strips. Use a strip for each side to make it stronger, leaving at least an eighth of an inch between the cards for easy bending.

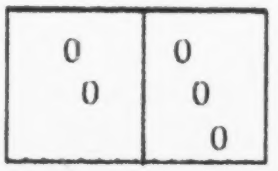
Using a paper cutter, cut the top card into one inch strips down to the center fold. With a magic marker print the numbers zero through ten on the lower card so that each number will be directly under a strip. Print Flip-It on the cover.





Number Families

Children make perception cards for number families relating addition and subtraction. Child picks a card, shows it to the class and selects another child to write one equation about the card. This child selects another, etc., until all four equations have been found.



2+3 = 5
3+2 = 5
5-2 = 3
5-3 = 2

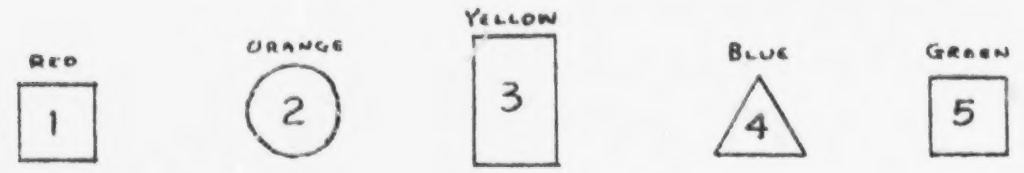
Flash Card Game

Place flash cards along the chalk ledge. Select two children to compete. The children should stand at opposite ends of the chalk board. On signal, they begin naming the answers. After he has named the correct answer, he may pick up the card and proceed to the next card. The child who names the most is the winner.

Flash-tabs

Flash-tabs, a series of posterboard numeral cards varying in shape and color, are recommended for use in pre-school, kindergarten, and primary grade activities. They are designed as a minipulative device for individual pupils. They may be used for recognition of numerals, shapes and colors, and may serve as models in writing the numerals.

Prepare the cards as shown. Circles are 5 inches in diameter. Squares are 5 inches. Triangles and oblongs are half of a 5-inch square. Print large numerals on both sides. Each child has his own set of cards which he arranges on his desk as shown. At first the younger children will need help from the teacher in arranging the cards so that the 5-inch squares frame the group of cards.



Activities with flash-tabs:

1. Expose a set or group of objects on the chalkboard or flannelboard. Have children show the numeral card which identifies the number in the set; the numeral card which represents one more than the number of the set; two more; one less; two less; all the numeral cards for numbers larger than the number of the set; all the numeral cards for numbers smaller than the number of the set.

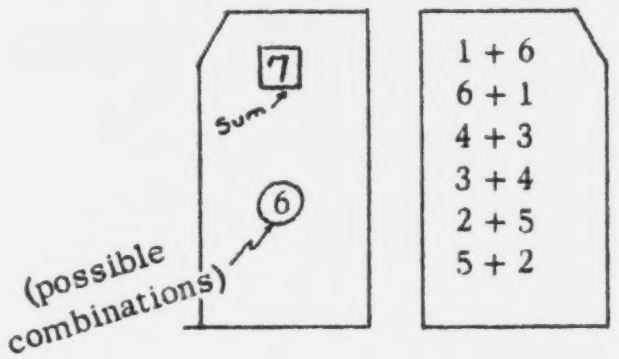
2. Display a numeral in the series 1-10. Have children show the numeral card for the number which comes just before the one represented; the one just after.

Flash-tabs (continued)

3. Have children read the numerals on the card just as they are arranged: Go to the right starting with the red card; with the purple card. Go to the left starting with the blue card; with the green card (also from top row to bottom row and from bottom row to top row). Name the numerals on the squares at the left; at the right; on the triangles; on the oblongs; on the circle.

Combinations

Child looks at one of 10 prepared cards, opposite sides of each card arranged as shown. The number in the box gives a sum. Child is to give all the simple number combinations that total this sum. He tries to give all such combinations as indicated by the circled number. (In this case, there are 6 combinations that can give the sum "7".)



Similarly, a set of cards can be made to indicate sums greater than 10, or products of 81 or less.

A simple sand-timer from the dime store will add interest--see how many cards can be finished in 1-3 minutes.

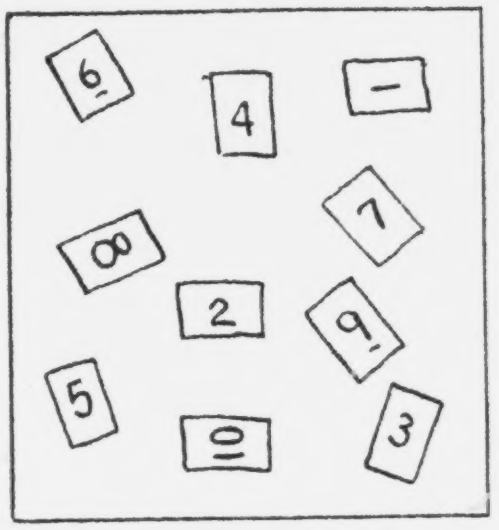
Opposite Operations

Materials: Large flash cards for addition, subtraction, multiplication, division. Child must give the opposite operation from that given on the card.

Example: If flash card is $4 + 3 = \square$, the child says, $7 - 3 = 4$ or $7 - 4 = 3$

Mind-Reader

Which card did you touch? Small cards with numerals 1-10 are scattered on a desk top. Student A mentally chooses a number, say 3. Student B points at random to any number he chooses until five of them have been pointed to. On the sixth, he points to "10", then on the seventh to "9", etc. Student A starts counting with the number he has chosen "3" and says this number to himself as B points to the first number. "A" continues counting until he reaches fifteen and says "STOP". The number Student B points to when Student A says "STOP" should be the number "A" chose in the first place. Why does this always work?



Around-the-Room

Materials: Flash cards (+, -, x)

Purpose: Give fast and correct answers to flash cards so that you may leave your seat and proceed around the room until you regain your original position.

Procedures:

1) Choose one person to stand and begin his attempt to go around the entire room (stopping at each student's desk) and back to his seat.

2) The person standing (selected), student "X", stands along the side of a person sitting, student "A", and the flash card is given. The two try to respond with the fastest correct answer possible. The person who wins proceeds or sits down.

   Example: Student "X" (standing) vs. student "Z" (sitting). Student "X" responds more quickly, he moves to the next person's desk.

   Example: Student "A" responds more quickly. He stands and student "X" takes his seat. Student "A" who is now standing tries to proceed as far as he can.

3) The student who finally reaches his seat again becomes the winner. (Student standing proceeds until he is defeated or declared the winner.)

Helpful rule: The person standing has one advantage: any responses that are ties--the person standing wins and moves on.

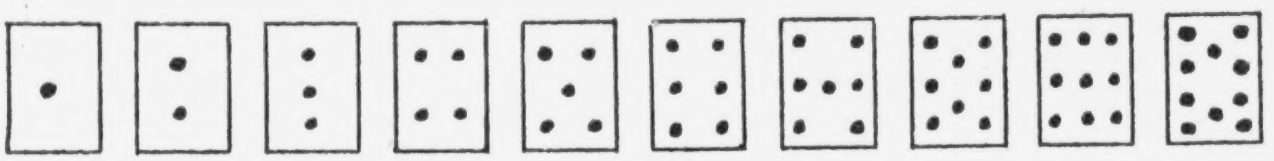
Comments: 1) You cannot predict the length of this drill game, (5-20 minutes), (probably best played once every week or two); 2) Advantage--many times to respond; 3) Wonderful opportunity to become a hero when you defeat the student who looks like the sure winner; 4) High and keen interest throughout the drill game.

Using the Tape Recorder

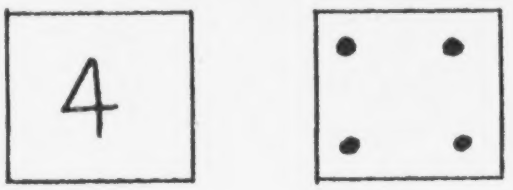
Record multiplication tables or any number facts on tape. Hesitate long enough for student to jot down his answer. Then give the correct answer. Problems can be used in the same way. This technique allows the student or students (using the jacks) to practice while the teacher is occupied with other students. (Students can dictate their own tape. ed.).

Domino Dots

For this activity, 20 cards (4 x 6) are needed. The numbers 1 through 10 are placed on ten cards, and on the other ten cards dots are placed:



1. Children count dots as group with teacher.
2. Children must go up to table and pick up card with the correct number of dots teacher asked for. (Sometimes children can be "teachers.")
3. Children count numerals as a group.
4. Child goes up to table and picks up card teacher asked for.
5. Child goes up to table and must pick up two cards which match.



Everybody Show

Cards with number combinations (but no answers) are passed to children. Teacher or leader asks, "Everyone who has another name for "seven" (etc.), show your card." All possible combinations for seven (1 + 6, 4 + 3, 8 - 1, etc.) are held up for others to see.

We have also made our own flash cards for the addition and subtraction facts. The children practice with each other. We made these with construction paper of different colors, but you may use oak tag or poster board.

Numbo

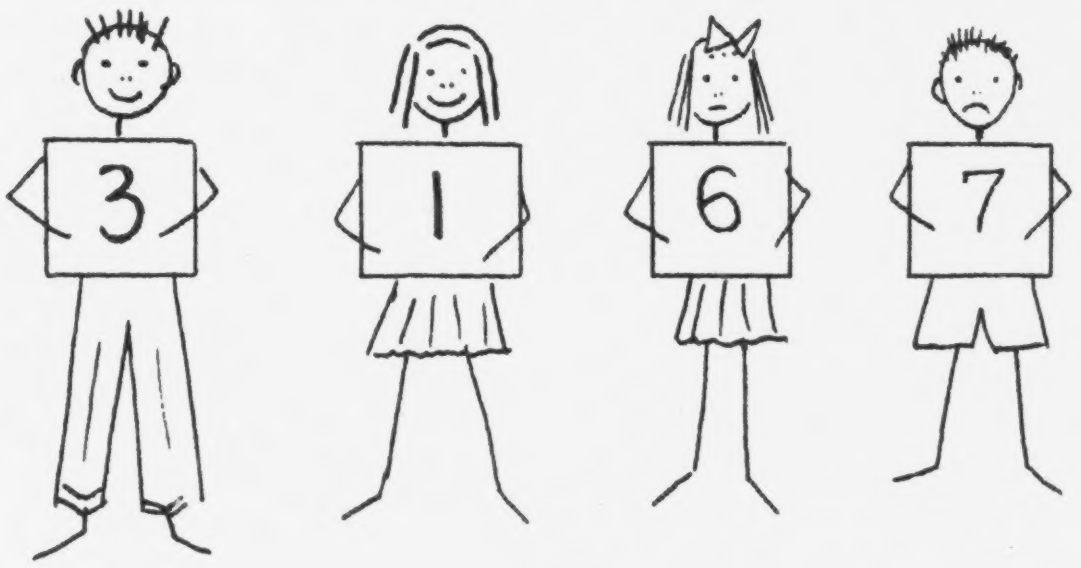
The game is played like Bingo. Cards made up with nine numbers (0-30). One child calls the number and puts it on the board. When someone fills a row, he calls out "Numbo." The winner then becomes the caller of the new number game. Cards are made from shirt boards.

0	3	17
20	8	19
7	16	14

Place Value

To help children understand place value and read large numbers--make one digit numerals on shirt cardboards. Pass out the cards. Ask two or more children to come to the front and arrange themselves facing class holding cards in front of them. The teacher then invites individuals or teams to read the number.

Children can be asked, "Who is in the ten's place?" or "Robert will move to the hundred's place."



Living Graphs

The children in the class form a type of bar graph (in this case the proper term probably is Histogram) as an introductory experience in graphing.

Preparation: Place the names of the twelve months across the wall or bulletin board.

Description: Each child places himself in the proper line according to the month of his birthday. The first child in each line stands directly in front of the name indicating the month of his birthday to keep a uniform starting place for each of the lines (bars).

Sample Questions:

A. Is there anyone in your month with the same birthdate?

B. Which month has the most birthdays in our class?

C. If you switch the order of people in your line, would some other month have the most birthdays?

D. Which month has the fewest birthdays?

E. What is the difference between the month with the most and the month with the fewest birthdays?

F. Did any months end in a tie?

G. Did anyone notice something interesting about our "living graph" he would like to tell the class?

Using a Grid

It can be difficult to carry around a "living" graph or even a stack of books especially if you want to show your graph to someone at home. Here is another way to make a bar graph.

Preparation: Draw a heavy dark line across the bottom of a grid marked off in one (1) inch squares. Below this line, number the vertical columns - 0, 1, 2, 3, 4, 5, 6, 7, etc. from left to right. Have crayons available.

Description: The children are asked to count the number of blocks from their house to school. Each child colors with a crayon one of the squares in the column telling the number of blocks he lives from school.

Example: If you live 5 blocks from school, you color a square in the column marked with the "5".

You might have each student write his name in the square he colored to demonstrate that his square represents the number of blocks he lives from school.

Sample Questions:

A. How many other people in this class live the same distance from school as you?

B. What can you tell from our graph?

LINUS' LINES

PURPOSE: Demonstrating the use of the metric and inch ruler. Metric ruler through tenths of a centimeter. Inch ruler through fourths of an inch. Estimating lengths to the nearest fourth of an inch and tenth of a centimeter. Ordering fractions with denominators of halves and fourths. Ordering decimals through tenths.

PROCEDURE: 2-4 players or total class activity. According to the length indicated on the playing card, the student will draw segments, using the inch or metric ruler. For rules of the game, refer to last page.

MATERIALS: 6 inch-centimeter ruler.

Student sheets.

Deck of playing cards: The "in." and "cm." must be written along with the numbers on the face of the cards. Each number listed represents one card.

cm.	cm.	cm.	in.	in.
.6	6.3	12.4	1/4	3 1/4
1	7	13	1/2	3 1/2
1.3	7.4	13.6	3/4	3 3/4
2	8	14	1	4
2.2	8.2	14.1	1 1/4	4 1/4
3	9	15	1 1/2	4 1/2
3.8	9.9		1 3/4	4 3/4
4	10		2	5
4.5	10.5		2 1/4	5 1/4
5	11		2 1/2	5 1/2

LINUS' LINES (Cont.)

cm.	cm.	cm.	in.	in.
5.1	11.7		2 3/4	5 3/4
6	12		3	6

Linus' cards: Write LINUS' CARD at the top and RETURN THIS CARD TO THE DECK AND SHUFFLE THE DECK below the instructions. Make 2 for the following instructions:

1. Measure the distance you need.
   Use 1 or 2 cards from your discard pile to make your segment.
2. Estimate the distance you need in centimeters.
   Construct the estimate.
3. Estimate the distance you need in inches. Construct the estimate.
4. Use either the smallest or largest centimeter card in your discard pile.
5. Use either the smallest or largest inch card in your discard pile.

OTHER: For a less difficult game, use only the whole number cards. Other variations of game may be used as teacher desires.

RULES: 2 - 4 players:

1. Each player needs a game sheet and a metric - inch ruler (minimum 6 inches.)
2. To begin play, each player draws a card at random the greatest length indicates player One. Play rotates clockwise. Shuffle deck.

LINUS' LINES (cont.)

3. A play consists of drawing a card and constructing a segment equal to the length indicated on the card. The first segment must start from the end of Linus' pointing finger, going in any direction. Successive segments must start at the end point of the last segment constructed, going in any direction. Player keeps the cards unless it is Linus' card.
4. The object of the game is to end a segment within circle 1, then continuing the segments, go on to circle 2, 3 and 4. The first player to end a segment within circle 4 wins the game.
5. A segment may not pass completely through any circle during the game. Segments may intersect.
6. Linus' Cards: Player drawing a Linus' card must follow the directions, construct his segment and return the card to the deck.

LINUS' LINES

4

18

1

20

3

2

Follow the Line

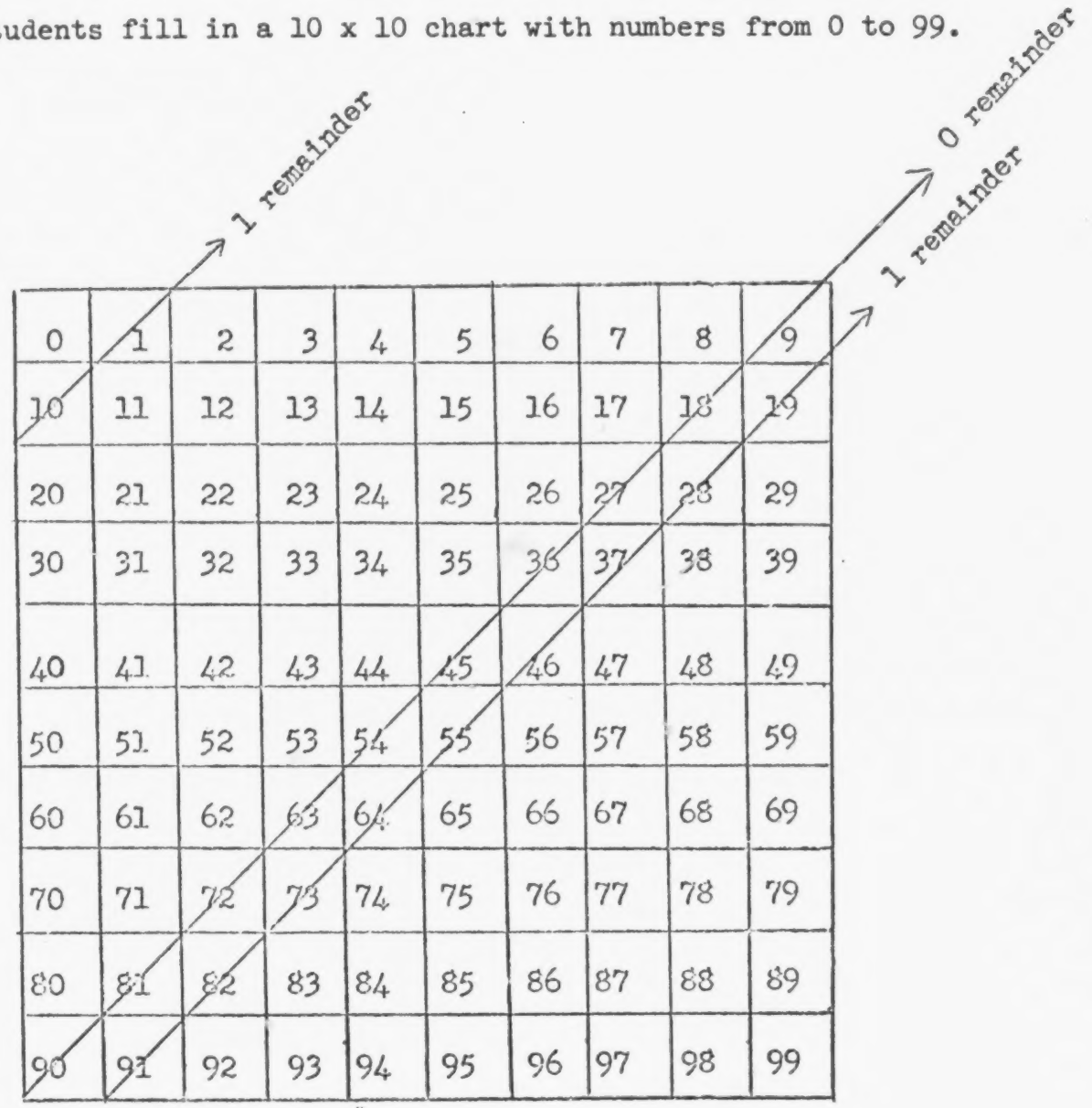
Ask the student the following questions and at the end of the game see if they can answer them.

Can you find all of the numbers between 0 and 99 that when divided by a certain number would all have the same remainders?

Can you do it without dividing?

It can be done!

Have the students fill in a 10 x 10 chart with numbers from 0 to 99.



0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Next have them list numbers divided by 9 with -

a remainder of {0 = 0,9,18,27, __, __, __, __, __, __}
a remainder of {1 = 1,10,19,28, __, __, __, __, __, __}
a remainder of {2 = 2,11,20,29, __, __, __, __, __, __}
a remainder of {3 = 3,12,21,30, __, __, __, __, __, __}

a remainder of {5 = 5, 14, 23, 32, __, __, __, __, __, __}

a remainder of {7 = 7, 16, 25, 34, __, __, __, __, __, __}

On their charts let them draw a line connecting the numbers together in each series.

Have them then find the pattern for numbers divided by 8 and draw lines connecting the numbers.

remainder {0 = 0, 8, 16, 24, __, __, __, __, __, __}

remainder {1 = 1, 9, 17, 25, __, __, __, __, __, __}

Students may use the same pattern for 7, 6, and 5 as well.

Not only are students using division concepts, but are reviewing multiplication, addition, and subtraction.

Pattern Spotting

Can you find all the number pairs that total 89?

Can you find any additional patterns that show interesting relationship between numbers?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Arrow Arithmetic

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1. 12→ tells you to start at 12 and move 1 space to the <u>right</u>. 12→ = 13
   26 → → → = 29

2. 12 ↓ tell you to start at 12 and move <u>1 space down</u>. 12 ↓ = 22
   26 ↓ ↓ = 46

3. 12← tells you to move 1 space to the <u>left</u> of 12. 12← = 11
   26 ← ← ← = 23

4. 12 ↑ tells you to start at 12 and move <u>up</u> 1 space. 12 ↑ = 2
   36 ↑ ↑ = 16

5. 12 ↘ tells you to start at 12 and move 1 space <u>down</u> and 1 space to the <u>right</u>.
   12 ↘ = 23
   36 ↘ ↘ = 48 (36 + 20 + 2)

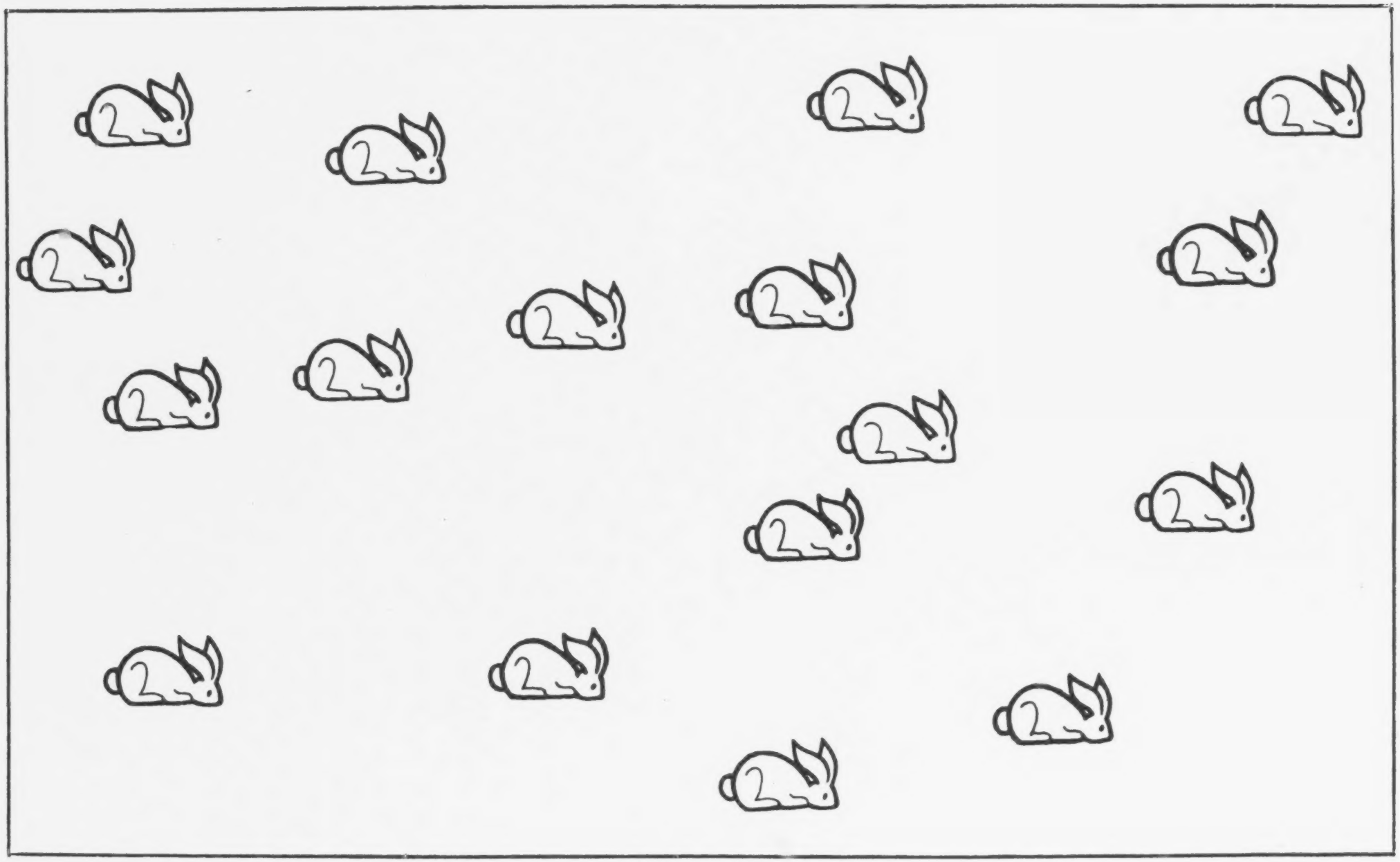
6. 26 ↖ tells you to start at 26 and move up 1 space and 1 space to the left of 26.
   26 ↖ = 15
   37 ↖↖ = 15

Use the number chart and find the answers to these examples:

9 → ↓ = _____
9 ↓ → = _____
37 ← ← ↑ = _____
37 ← ↑ ← = _____
32 ↗ → ↑ = _____
45 ↗ → ↑ = _____
68 ↗ → ↑ = _____

17 → = _____
17 ↓ → = _____
24 ↓ ↓ → = _____
24 → ↓ ↓ = _____
36 ← ↓↓ → ↗ = _____
56 ← → ↗ ↓↓ = _____

PEN THE BUNNIES

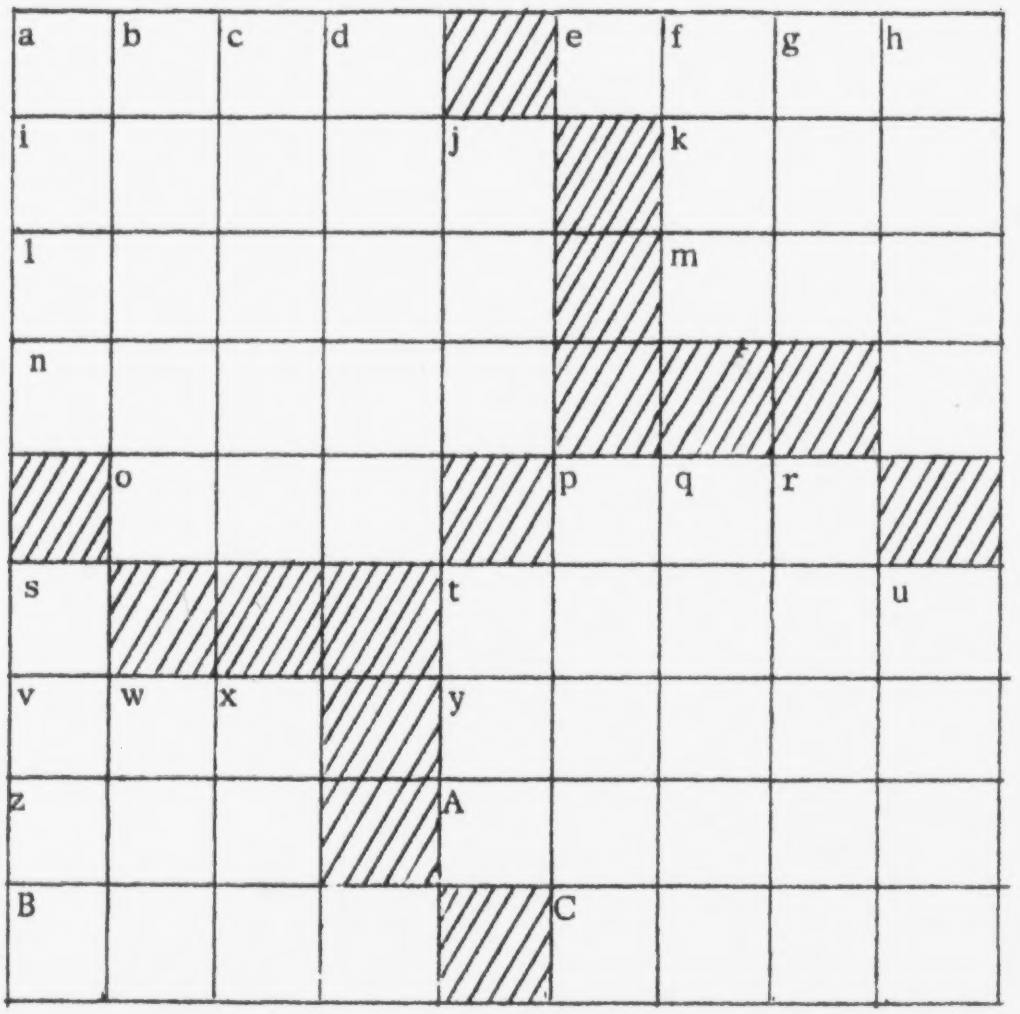


Draw six straight lines that will separate each of the bunnies.

Follow the Dots

1. 2 x 9	15. 2 x 5	29. 1 x 3	43. 7 x 11
2. 9 x 9	16. 9 x 10	30. 7 x 8	44. 3 x 3
3. 4 x 2	17. 5 x 1	31. 4 x 9	45. 10 x 10
4. 11 x 6	18. 12 x 5	32. 4 x 3	46. 12 x 12
5. 4 x 8	19. 11 x 9	33. 7 x 9	47. 2 x 1
6. 10 x 7	20. 8 x 10	34. 11 x 3	48. 11 x 5
7. 8 x 9	21. 7 x 3	35. 10 x 4	49. 3 x 9
8. 5 x 5	22. 6 x 5	36. 1 x 1	50. 2 x 2
9. 1 x 7	23. 11 x 8	37. 11 x 11	51. 4 x 5
10. 2 x 11	24. 8 x 8	38. 5 x 9	52. 12 x 10
11. 7 x 12	25. 2 x 3	39. 4 x 4	53. 6 x 8
12. 7 x 5	26. 4 x 6	40. 11 x 12	54. 7 x 4
13. 8 x 12	27. 9 x 12	41. 5 x 10	
14. 6 x 9	28. 6 x 7	42. 1 x 0	

Addition and Subtraction Puzzle



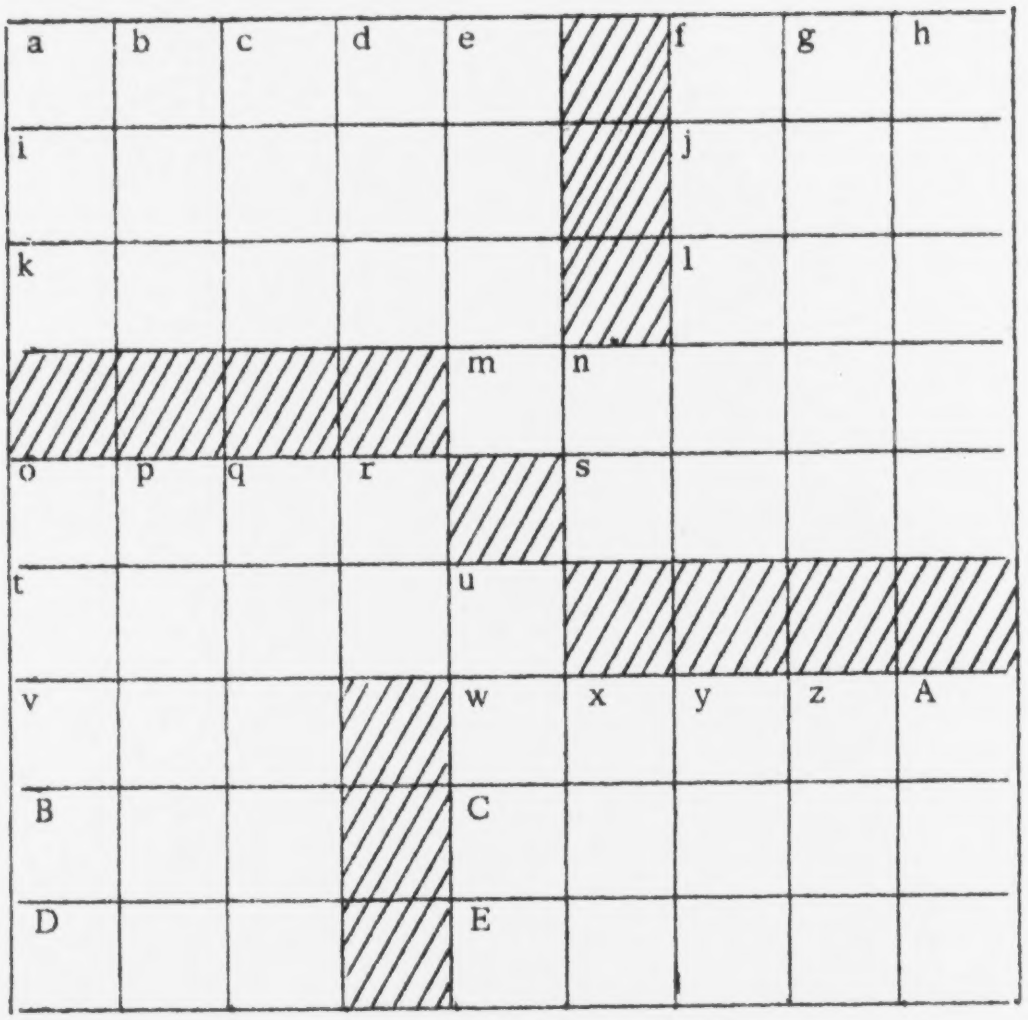
Across

a.) 7, 469 - 5, 347
e.) 17, 143 - 11, 856
i.) 5, 600 + 7, 048 + 9, 875 + 7, 761
k.) 19, 326 - 18, 893
l.) 8, 217 + 6, 543 + 9, 050 + 2, 704
m.) 2, 160 - 1, 876
n.) 7, 586 + 5, 782 + 8, 777 + 4, 328
o.) 5, 243 - 4, 247
p.) 210 - 87
t.) 3, 725 + 5, 806 + 7, 625 + 7, 861
w.) 700 - 256
y.) 9, 729 + 3, 685 + 8, 762 + 4, 491
z.) 1, 502 - 834
A.) 5, 430 + 2, 429 + 9, 052 + 8, 217
B.) 4, 073 - 2, 853
C.) 8, 724 - 319

Down

a.) 6, 211 - 3, 889
b.) 2, 517 + 2, 429 + 4, 703 + 1, 020
c.) 4, 917 + 6, 659 + 3, 785 + 7, 188
d.) 5, 190 + 9, 478 + 8, 363 + 5, 145
f.) 4, 001 - 3, 759
g.) 6, 973 - 6, 135
h.) 2, 562 + 1, 754 + 1, 487 + 1, 537
j.) 7, 300 - 6, 857
p.) 5, 067 + 1, 638 + 7, 283 + 1, 670
q.) 1, 889 + 6, 245 + 7, 856 + 4, 624
r.) 8, 265 + 8, 496 + 7, 046 + 7, 813
s.) 21, 003 - 19, 542
t.) 1, 670 - 1, 448
u.) 2, 401 + 1, 924 + 1, 538 + 1, 922
w.) 1, 954 - 1, 492
x.) 704 - 222

Multiplication and Division Puzzle



Across

a.) 63 x 549

f.) $35\overline{)27825}$

i.) 35 x 768

j.) $124\overline{)58652}$

k.) 83 x 425

l.) $750\overline{)144000}$

m.) 27 x 1309

o.) 25 x 367

s.) $60\overline{)74700}$

t.) 69 x 567

u.) $88\overline{)10648}$

u.) 58 x 442

B.) $95\overline{)21185}$

C.) 17 x 1463

D.) $39\overline{)30654}$

E.) 26 x 2189

Down

a.) $51\overline{)16473}$

b.) $17\overline{)7905}$

c.) $26\overline{)15132}$

d.) $107\overline{)94909}$

e.) 3 x 2351

f.) 43 x 1724

g.) 6 x 16324

h.) 65 x 819

n.) $348\overline{)17748}$

o.) 23 x 4049

p.) 46 x 418

q.) 9 x 7904

r.) $435\overline{)22620}$

u.) 43 x 75

x.) $77\overline{)42042}$

y.) $117\overline{)80613}$

z.) $60\overline{)22260}$

A.) $207\overline{)127098}$

Answers to Multiplication and Division Puzzle

2	1	2	2		5	2	8	7
3	0	2	8	4		4	3	3
2	6	5	1	4		2	8	4
2	6	4	7	3				0
	9	9	6		1	2	3	
1				2	5	0	1	7
4	4	4		2	6	6	6	7
6	6	8		2	5	1	2	8
1	2	2	0		8	4	0	5

Answers to Addition and Subtraction Puzzle

3	4	5	8	7		7	9	5
2	6	8	8	0		4	7	3
3	5	2	7	5		1	9	2
				3	5	3	4	3
9	1	7	5		1	2	4	5
3	9	1	2	3				
1	2	1		2	5	6	3	6
2	2	3		2	4	8	7	1
7	8	6		5	6	9	1	4

Tangrams

The Tangram is an ancient Chinese puzzle. It is perhaps the oldest and most enduring of geometric puzzles. It consists of a square divided into seven geometric shapes: two large triangles, a medium triangle, two small triangles, a square, and a rhomboid. A great number of geometric and pictorial arrangements can be made with these seven pieces.

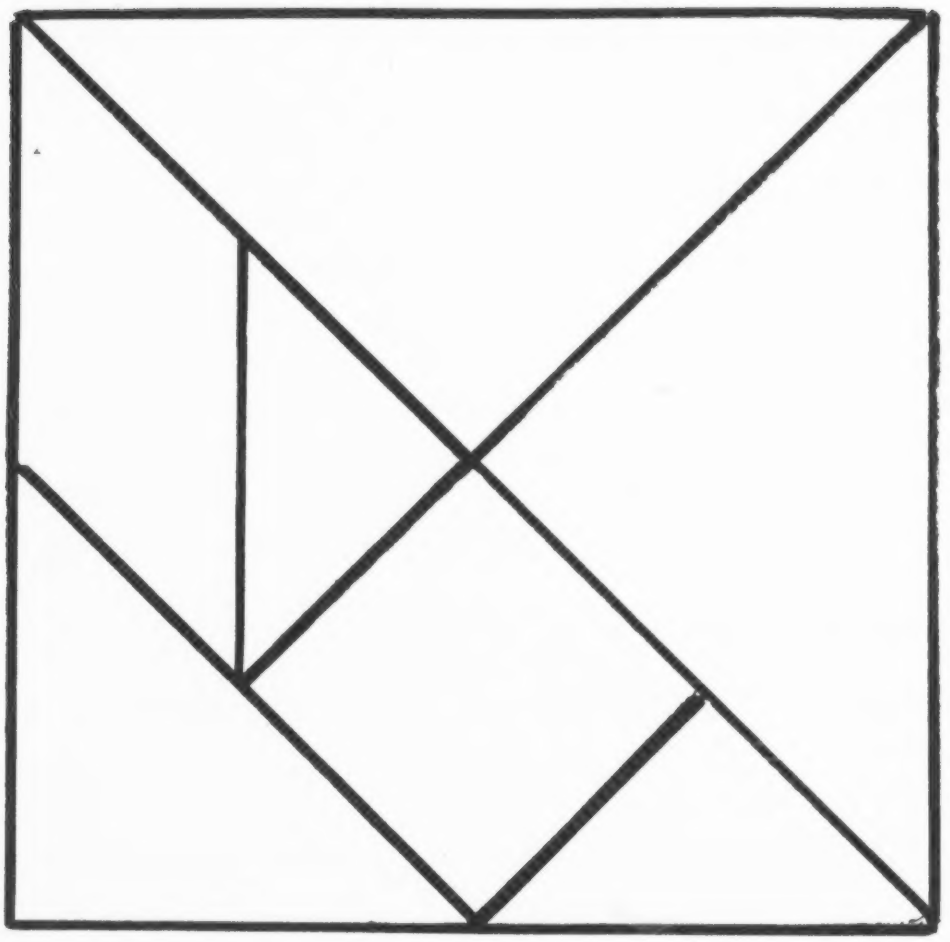
Begin to use the Tangrams by allowing the children free choice with the seven pieces. Allow them to see how the pieces fit together and what designs they can make.

Other exercises and challenges:

1. Trace around the seven pieces and make your own design on a piece of paper.
2. Provide children with pattern or task cards; place the pieces on the table so that they dorm the pattern on the task card. When using this activity with younger children, provide task cards which the pieces will fit within. (The patterns on the next page are small for the purposes of providing you, the teacher, with many suggested patterns...and the solutions to same!)
3. Have the children make their own Tangram set. Don't give them a ditto! Its a great paper folding exercise; lots of measurement without a ruler!
4. Challenges: Can you use all seven pieces to make a square?
   Can you make a rectangle that is not a square?
   Will the seven pieces make a triangle? A rhomboid?
   What letters can you form using all seven pieces?
5. Discover relationships among the pieces: Can you construct a large triangle from any of the five pieces which are left? How? Can you construct a square from any of the pieces left? How? Which pieces can you construct with the remaining pieces? If the value of the square equals 1, what is the value of the remaining pieces? If the square equals 2? 3? 10?

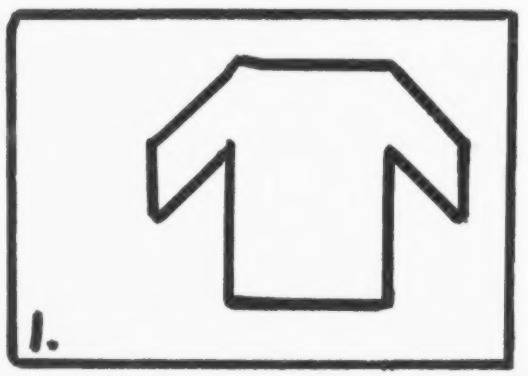
(See next page)

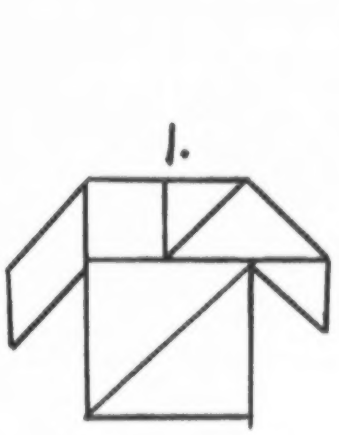
The Tangram Puzzle



These are solutions to Tangram Task Cards (made by teacher). The Task Card given to the child should include the outline only and the outline should be such that the actual Tangram pieces fit within the outline.

This is a minature version of the Tangram Task Card:

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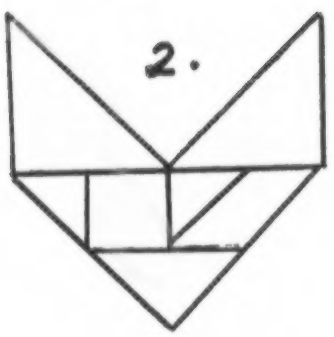
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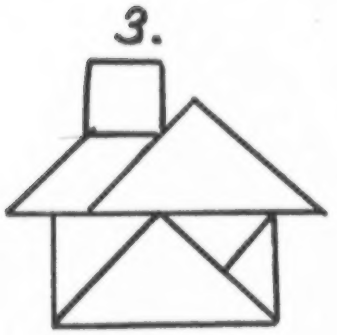
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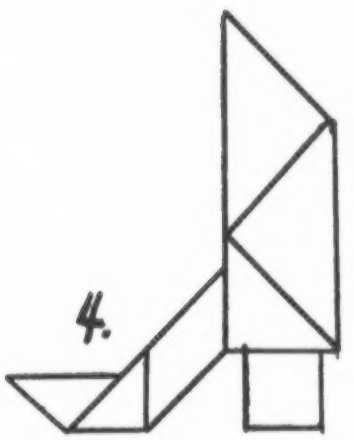
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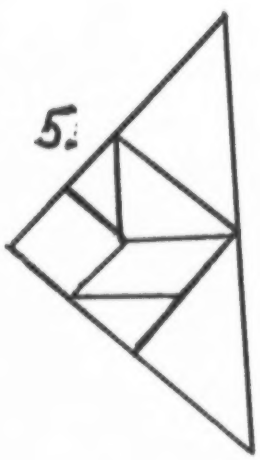
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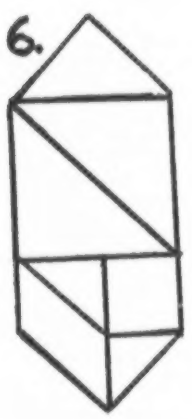
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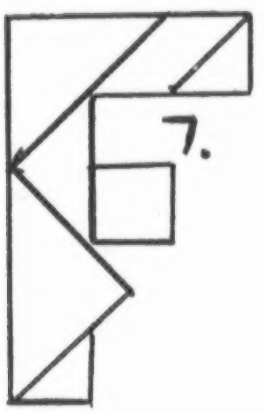
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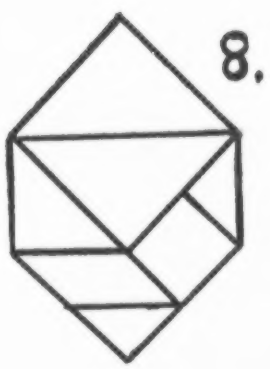
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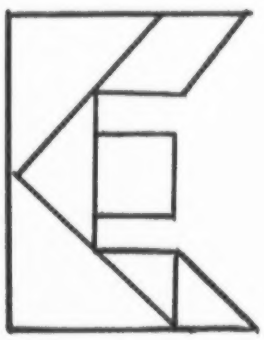
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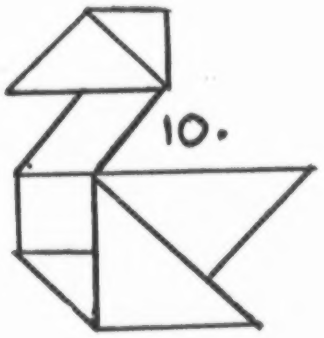
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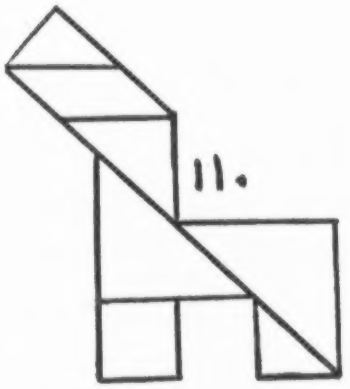
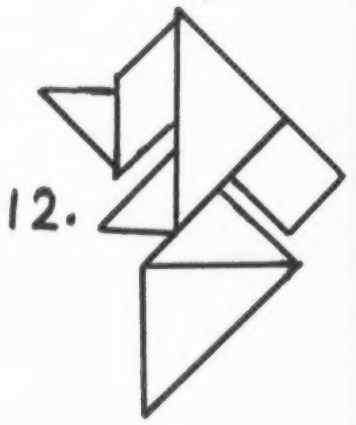

11.


12.

Secret Number Sentence

Here is a game for primary arithmetic, providing stimulating drill on mathematical sentences and on addition, subtraction, multiplication, and division combinations.

Player A secretly fills in this pattern with a number sentence:

□ ○ △ = _____

Player A must choose his numbers from some agreed-upon set. In Grade 2, a suitable set would be 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. If he selects 6 + 3 = 9 as his secret number sentence, he replaces the square and triangle with 6 and 3, the circle with the operational sign +, and the blank with 9. It must also be understood which operations are to be used.

Player B puts the following on his paper:

□	○	△	= _____
0	+	0	0
1	-	1	1
2		2	2
3		3	3
4		4	4
5		5	5
6		6	6
7		7	7
8		8	8
9		9	9

Then Player B proceeds to ask certain questions of Player A until he discovers what Player A's secret sentence is. The object is to discover the secret sentence in as few questions as possible. Player B can only ask the following questions:

1. Is the numeral in the (square, triangle, blank) equal to ?
2. Is the numeral in the (square, triangle, blank) greater than ?
3. Is the numeral in the (square, triangle, blank) less than ?
4. Is the operation ? (addition, subtraction, etc.)

Each time Player B asks a question about a position, he is able to eliminate one or more numerals under that position. For the secret sentence 6 + 3 = 9, the question, "Is the numeral in the square equal to 4?" would eliminate 4 as a possibility for the square. And the question, "Is the numeral in the square greater than 7?" would be answered "No," revealing that 8 and 9 are not possible. After these two questions, Player B's paper should look like this:

Secret Number Sentence (continued)

□	○	△	=	____
0	+	0		0
1	-	1		1
2		2		2
3		3		3
4		4		4
5		5		5
6		6		6
7		7		7
8		8		8
9		9		9

If his next question were, "Is the numeral in the square less than 5?" he would get a response of "No," and cross off 0, 1, 2, and 3. Then one or two more questions will reveal that 6 is the numeral in the square.

As the game is played, the pupil should become aware that "greater than" and "less than" questions usually yield more information than "equal to" questions.

When Player B succeeds in discovering Player A's secret sentence, the roles are reversed and Player A becomes the questioner. The player who discovers the other player's secret with the fewest questions wins.

It is advisable to start the game with the teacher in the role of Player A and the teacher in the role of Player A and the children asking questions, and to proceed in this manner until the children know how to play. However, while the game is highly stimulating as a group activity, it is doubtful that the group approach will result in significant gain for the child who needs it most. The kind of thinking promoted by the game requires that the child have time to ponder--to decide which questions to ask, and what information an answer provides him.

In the upper grades the game can be made more challenging by including all four fundamental operations and by allowing the relationship to include "greater than" and "less than" as well as "equals," so that secret sentences such as $4 \times 8 < 380$ may be used.

Secret Code

2	7	4
5	6	9
8	3	1

1. ⌊ ⌋ + ⌈ ⌉ = 10

2. ⌈ ⌋ + ⌈ ⌉ = 17

3. ⌊ ⌋ x ⌊ ⌋ = 20

4. ⌈ ⌉ ÷ ⌊ ⌋ = 3

5. ⌊ ⌋ x ⌈ ⌉ ÷ ⌊ ⌋ = 6

6. ⌈ ⌋ - ⌊ ⌋ = 4

7. ⌈ ⌉ - ⌊ ⌋ = 4

8. ⌈ ⌉ + ⌈ ⌉ = 6

Look at the above picture carefully. Notice the two vertical and two horizontal lines. These lines form a unique kind of "frame" around each number. The teacher then makes up some secret code problems using only the "frames." The children develop perception and the ability to do mental arithmetic.

Magic Squares

When a child correctly fills in the simplest magic square, his work is only beginning. The solution to this magic square usually arouses his interest in other magic squares and the patterns that may be found. From this first magic square, a child is able to discover an unlimited amount of other ones.

8	1	6
3	5	7
4	9	2

9	2	7
4	6	8
5	10	3

4	-3	2
-1	1	3
0	5	-2

The simplest way to form a new magic square is to add or subtract the same number to or from the existing numbers. This has been done in the second and third squares above.

To find a magic square using the numbers 3 - 11, add 3 + 11. Divide the result by 2. The 7 is in the middle square. Multiply 7 by 3 to find the total of the rows. In general terms, this is stated: Add the first and last number of the series and divide by 2 to get the center number. The center number multiplied by 3 gives the totals of the rows and columns. This works only for numbers in a series such as 5 - 13 or 100 - 108.

Another pattern the children readily see is, if the center number is odd, the corners are even and if the center number is even, the corners are odd. If correctly done, the even or odd numbers should form a plus sign in the center of the magic square.

Given only one fact, such as the total of the rows, or the center, or the last number in the series, the children find it quite easy to make a magic square.

More Magic Squares

Besides the magic squares shown on the first page, there are also subtraction magic squares. Instead of adding these numbers, add only the two end numbers in each row, column, or diagonal and subtract the center number in that row. For example, in the first row across, add 2 + 4 to get 6. Subtract the middle number of that row, 1, and the answer is 5. In each case in this magic square, the answer is always 5. Can you add or subtract the same number in this magic square and get a new one?

2	1	4
3	5	7
6	9	8

This is a multiplication magic square. For this square, the numbers must be multiplied. The total for this square is always 216. Can you find new magic squares by adding or subtracting a number?

12	1	18
9	6	4
2	36	3

Fractions can also be used in magic squares. Fraction magic squares have the same properties as the magic squares shown on the first page. It is very easy to get a new magic square from an old one.

$\frac{7}{8}$	$\frac{7}{16}$	$\frac{3}{4}$
$\frac{9}{11}$	$\frac{11}{16}$	$\frac{13}{16}$
$\frac{5}{8}$	$\frac{15}{16}$	$\frac{1}{2}$

Magic Squares

(Sum of each row, column and diagonal is 15.)

	1	
	5	
4		2

4		8
2		6

	9	
3		7
	1	

The basic magic square is this:

8 1 6
3 5 7
4 9 2

An infinite number of magic squares may be made from this basic one by:

1. leaving 5 in the center and rotating the other numbers, so that keep the numbers in order and move all in the same direction 2, 4, or 6 spaces;

2. adding a constant to each number (i.e. 3) subtracting a constant from each number;

3. multiplying each number by a constant (i.e. 5) or dividing each number by a constant. Here is the basic one after each number has been divided by 2:

$$\begin{array}{ccc} 4 & \frac{1}{2} & 3 \\ 1\frac{1}{2} & 2\frac{1}{2} & 3\frac{1}{2} \\ 2 & 4\frac{1}{2} & 1 \end{array}$$

Which number spoils this Magic Square?

27	6	21
11	18	24
15	30	9

P	N	D
18	0	0
13	1	0

Pennies
Nickles
Dimes

How many different ways can I have 18¢ in my pocket?

The Upside-Down Magic Square

The magic square shown directly below can be held upside-down and a new magic square appears. Counting the regular and the upside-down magic square, there are about 48 ways to make a total of 264.

96	11	89	68
88	69	91	16
61	86	18	99
19	98	66	81

I X O H O X I

The unusual name of this magic square is a clue to the unusual properties of this magic square. The name is the same backwards as forwards, upside-down as right side up, in the mirror upside down as well as right side up. No matter how you look at the name, it is always the same.

The same is true for the magic square. If the square is held upside-down, in the mirror, or upside down in the mirror a magic square appears. There are more than 100 ways to make a total of 19998.

8818	1111	8188	1881
8181	1888	8811	1118
1811	8118	1181	8888
1188	8881	1818	8111

Crossword Magic Square

1. Used to tie up packages (6 letters). Place the letters in the following squares: 14, 9, 15, 11, 2 and 5
2. Used to write on a blackboard (5 letters). Place the letters in the following squares: 18, 3, 13, 7 and 12
3. A mad, noisy, unruly crowd (3 letters). Place the letters in the following squares: 6, 1 and 10
4. Past tense of feed (3 letters). Place the letters in the following squares: 17, 4 and 16
5. Abbreviation for Justice of the Peace (2 letters). Place the letters in the following squares: 8 and 20
6. The only letter in a word that is always followed by U. Place in square 19.

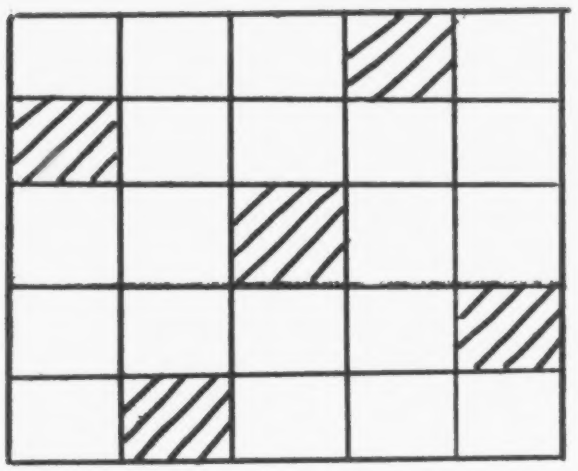
1	2	3		4
	5	6	7	8
9	10		11	12
13	14	15	16	
17		18	19	20

If you have done this correctly, the diagram will not contain any words and each square will contain a different letter.

Now place the number of each letter as it appears in the alphabet (A = 1, B = 2, C = 3, etc.) in the correct square below. For example, square number 6 above should contain M, so you would write 13 in the same square below. (M being the 13th letter in the alphabet.)

You should now have a magic square that totals 42. There are many ways to get a total of 42 from four numbers in the diagram below. How many can you find?

See if you can find 25.

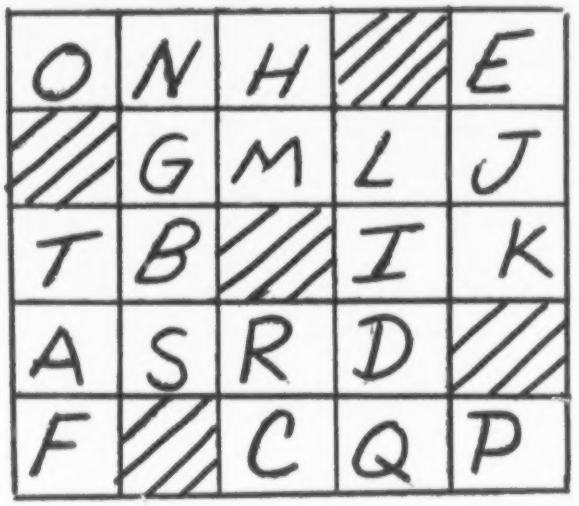


* * * * * * * * * * * * * * * * * * * *

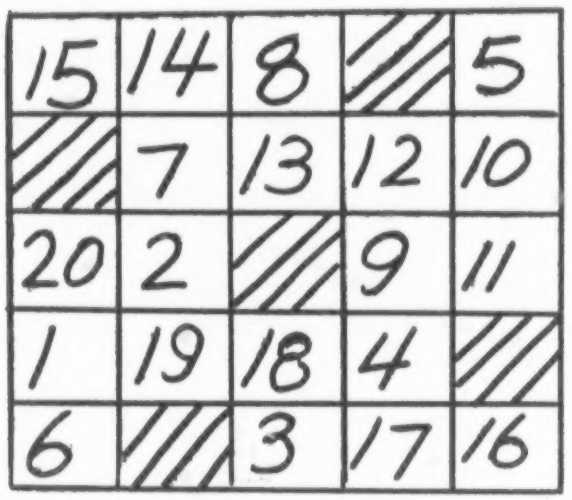
SOLUTION:

If you have correctly filled in the crossword, it should look like the one below.

The correct answers are: 1. string 2. chalk 3. mob
4. fed 5. J.P. 6. q



If you have correctly changed the letters into numbers, your diagram should look like this:

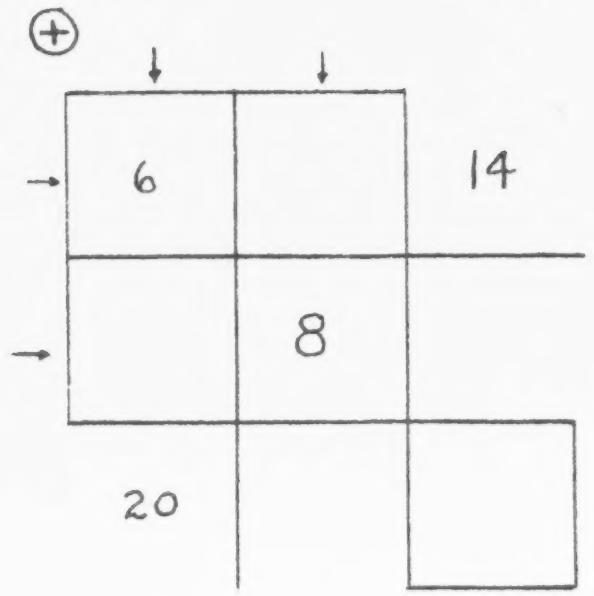


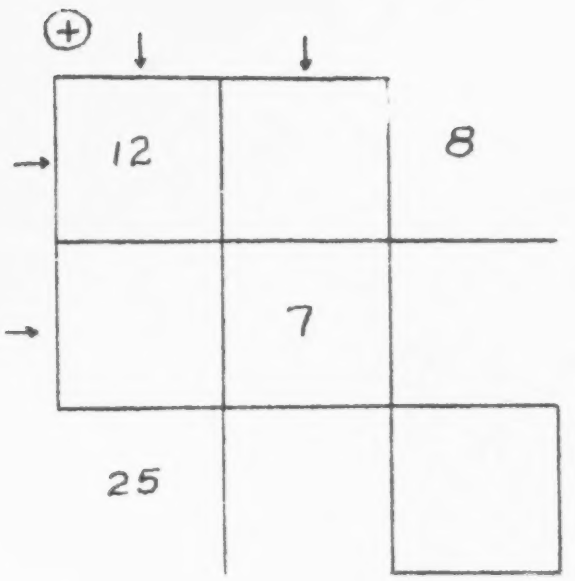
SOLUTION (CONT.)

Some of the possible number combinations that total 42 are:

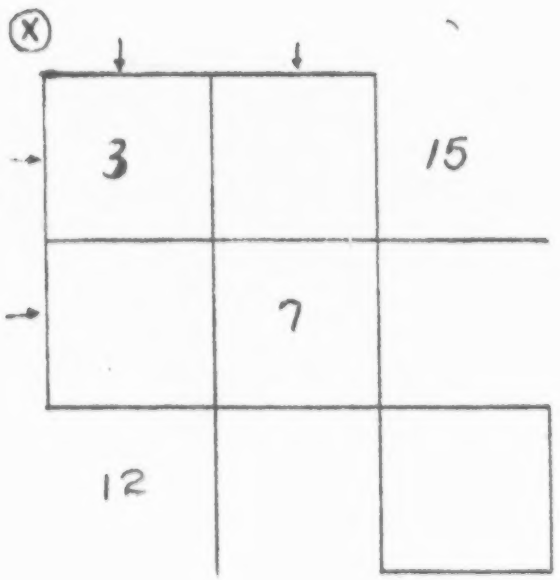
five groups vertical	12, 10, 9, 11
five groups horizontal	18, 4, 3, 17
two diagonals	20, 1, 2, 19
the four corners	14, 8, 7, 13
8, 11, 3, 20	10, 17, 1, 14
20, 1, 10, 11	7, 12, 19, 4
13, 9, 2, 18	12, 2, 19, 9
14, 8, 3, 17	17, 4, 14, 17
1, 19, 12, 10	18, 4, 7, 13
20, 6, 5, 11	16, 10, 1, 15
3, 16, 15, 8	14, 5, 6, 17

Criss-Cross





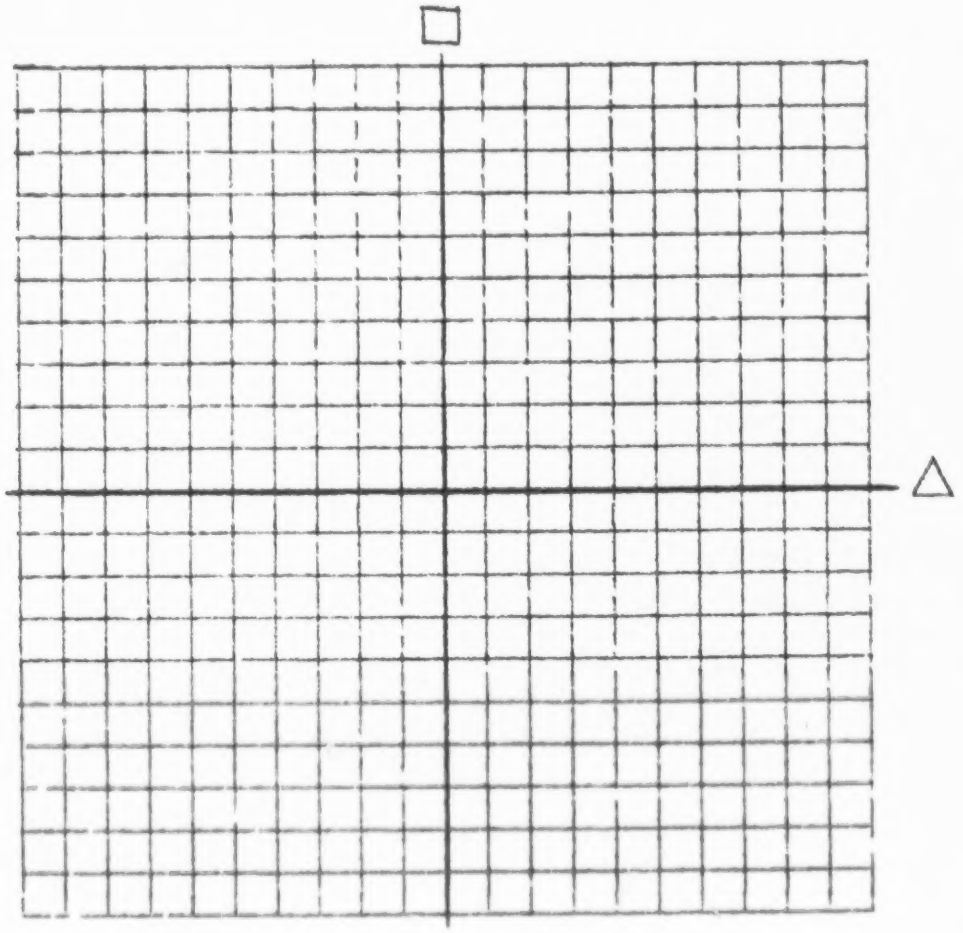
Children fill in missing addends, then complete the missing sums. The final check is to reach the "grand slam" in the corner box.



Multiplication is done in the same manner. However, the student will have to know compound multiplication, or the problems will have to be very simple.

Tic-tac-toe

The game is played with the entire class. An overhead projector with a prepared grid makes things easy. However, a grid drawn on the board will do just as well.



The instructions for the game are purposely short and simple. It will be up to the participants to fill in the gaps.

The class is divided into two teams (e.g., boys versus girls). The teams do not have to be of equal number or ability. One pupil is assigned the job as a recorder. He or she may or may not participate in the game. His job is to record, in two columns on the board, the numbers that are given to them by the students.

	△	□
x	2	3
0	4	1
x	3	0

(continued on next page)

Tic-tac-toe (continued)

The teacher then gives the following instructions to the students:

1. This is a game of tic-tac-toe, but in this game in order to win you must get five "X" or "O" in a row.
2. In order to tell me where to place your "X" or "O" you must give me two numbers. Each number must be equal to or less than 10. The recorder will write these numbers on the board and I will place your "X" or "O" in the correct place on the grid. Watch me closely and see if you can understand how I place them.
3. Once you say a number you may not change your mind. Think before you tell us your numbers, but if you take too long you will lose your turn.
4. You are not allowed to help your team members. (This rule can be altered at the teacher's discretion.)

The game follows the rules of coordinate axes with the students supplying the two variables. The teacher should be careful that he or she does not count by pointing to the lines but rather simply placing the marks in the correct place.
The first couple of games played will most likely be played and won all in the first quadrant.
In order to force the game out of the first quadrant, the teacher may reduce the limits set in rule #2 to "numbers equal to or less than five." The game will then quickly come to a stalemate with the first quadrant completely filled in. At this point the students will urge the teacher to tell them how they can get out of the first quadrant.

Facts Practice

An X and O or Tic-Tac-Toe board is put on the floor with masking tape. The group is divided into two teams, the X's and O's. A fact is asked of one person and if he answers it correctly, he can go stand on one square of the board. A person from the opposite team is asked next, and if he answers correctly, he can stand on the board. This continues until 3 X's are in a row or 3 O's (or "cat" wins). The children really seem to enjoy this.

ic
Nu-Tac
oe

The game described here is a type of tic-tac-toe game that has ingredients of strategy common to checkers and chess: timing, blocking, position, anticipation. Moreover, children, as well as adults, learn this game in less than one minute, get to grips with the challenge very quickly, and seem to derive considerable enjoyment in playing it.

1. Start with four pieces marked with crosses and four pieces marked with circles, and arrange them as shown on the twenty-square board in Figure 1.

2. Two players play. One moves the pieces with crosses, the other moves the pieces with circles, taking turns, first one player then the other.

3. A move consists of pushing your own piece into an adjacent vacant square up or down, to the right or to the left, but NOT diagonally. There is no "jumping" or "taking" in this game, and if a square is occupied, no other piece may be moved into this occupied square.

Figure 1

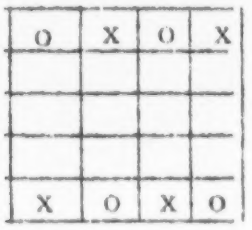


Figure 2: Examples of wins for the circles.

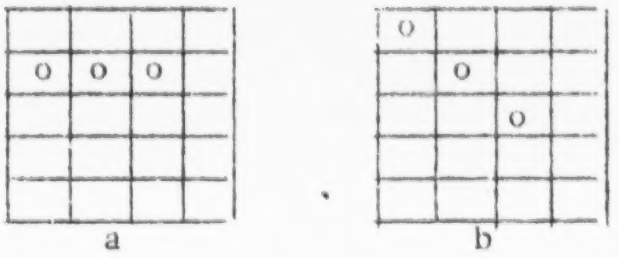
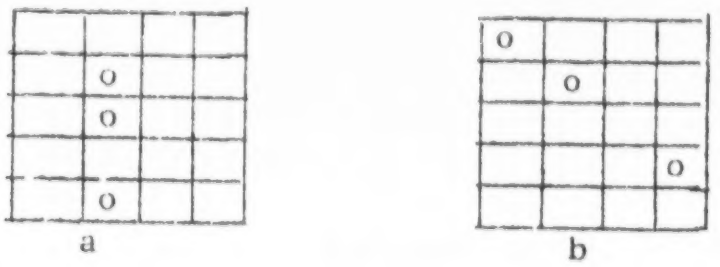


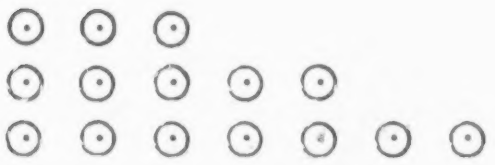
Figure 3: Examples of no wins for the circles.



4. The object of the game is to place three of your pieces in the same line; vertically, horizonatlly, or diagonally without any intervening vacant square.

Variation: One player places all eight pieces on the board into squares in any arrangement he chooses, one to a square. The other player makes the choice of moving first or second.

Nim (A Game of Strategy)



Take one or more from one and only one row.
The person who takes the last counter wins.

Let the students discover their own strategies.

A Computer Game

Choose any natural number. If it is even, divide by 2. If odd, multiply by 3 and add 1. Continue the process and see what happens.

Example: 7 → 22 → 11 → 34 → 17 → 52 → 26 → 13 → 40 → 20 → 10 → 5 → 16 → 8 → 4 → 2 → 1.

Will you always come down to 1?

Computer

Child volunteers to be the "Computer." A series of mental computations are presented to him rapidly. He continues until an error is made and detected by another student who raises his hand so that computation stops. If that student can correct the "Computer's" error, he becomes the new computer. One child can be named the "counter" to keep track of the number of problems successfully completed by each "Computer." Difficulty of computation and number of processes involved increase with grade level. First graders may stay with sums to 10 and the related differences. Sixth graders may use all processes.

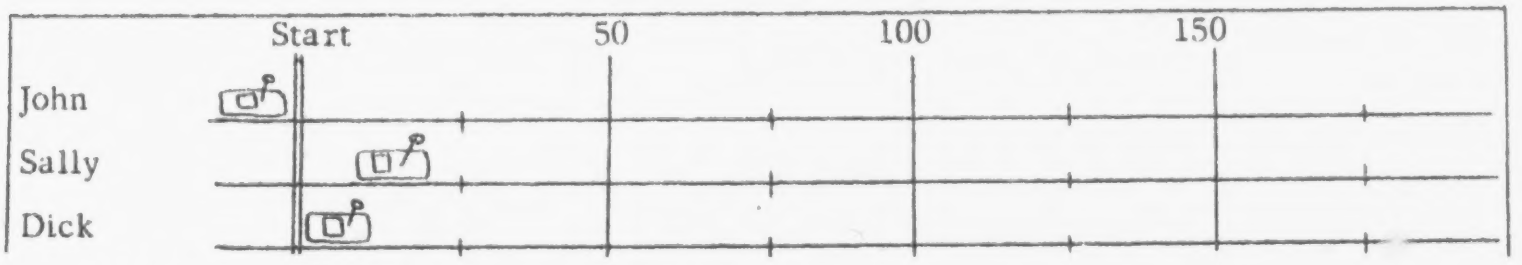
The Great Race

This activity takes some preparation but can go on for weeks.

Each day the class is presented with five computation problems given orally by the teacher. Each child writes his answers on small sheet and finds the sum of these answers. One point is given for each correct answer, five points for a correct sum. Highest possible score each day is 10.

Problem	Answer	
1	12	C
2	11	C
3	20	C
4	3	✓
5	4	C
Sum:	50 ✓	Points (4)

On a large sheet of poster board, mounted on a tack board:



Each child has a track and a marker (inexpensive plastic car held to track with a pin). Once every week or so, children find their accumulated points since the last race and move their markers a corresponding distance on the track. Leaving a trail of colored magic marker each time helps them see if they have improved their speed and accuracy. They "race" against themselves--the successive colored trails tell the story.

The teacher may vary the process, difficulty of problems, and timing from one day's "heat" to another as class seems able to respond.

Note: Moving his marker is the responsibility of each child to do during "free" time. The whole daily experience should take less than five minutes.

Numeral Game

Many children need practice in naming the numerals. Divide the class into three groups and choose one person from each group to point to a numeral on the number line. This person chooses someone from their group to name the numeral. If the child answers correctly, he gets to be the next pointer for his group.

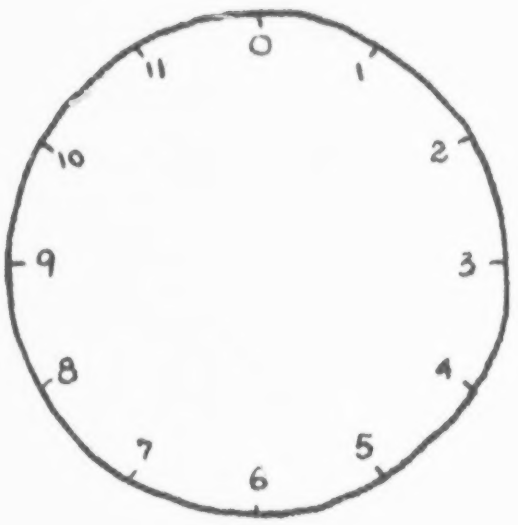
The Last Straw

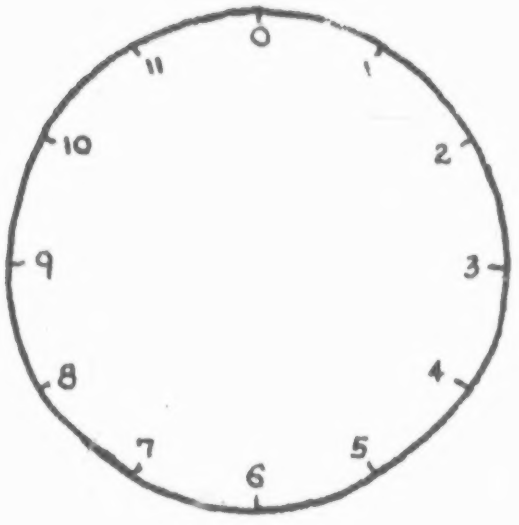
Two players. Fifteen pieces (2") of plastic drinking straws are tossed on a desk top. Object of the game is to force your opponent to pick up the last straw. Each player, at his turn, may pick up 1, 2, or 3 straws. Can you figure a way so that you will always win?

Button Game

To play this game, assemble a paper cup, a paper plate, and six to ten buttons (two or three different colors will facilitate the recognition of groups). Glue the paper cup exactly in the middle of the paper plate and allow it to dry thoroughly. Two to four children play. The players take turns throwing the buttons one at a time into the paper cup. The exact distance to stand from the plate is determined, measured accurately, and marked. Scoring for this game is done according to the desired scoring device. Each child playing will determine these facts: "How many did I throw into the cup?" "How many did I miss?" "Do the ones in the cup and the ones outside equal the total number I had to begin with?"

Clock Game



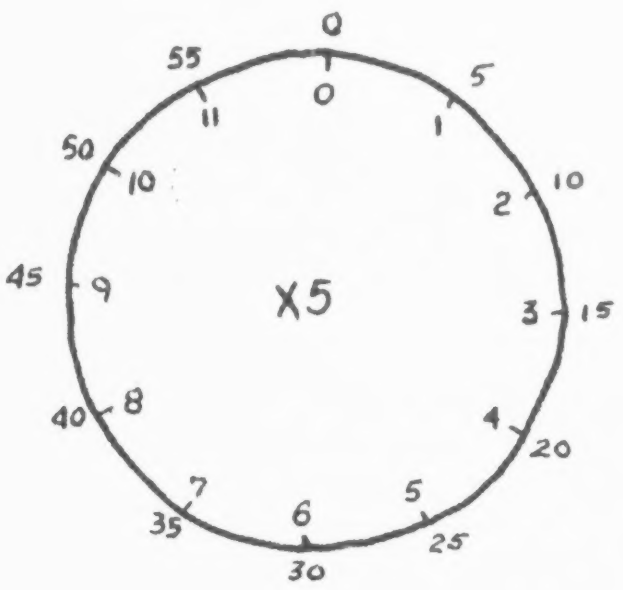


Players: Two Teams

Each team sends a player to the board. He stands with a piece of chalk facing the class. The teacher writes the same multiplier in each circle. The teacher then says, "Go." Students turn around and writes the answer outside the circle. For example: the center number is 5, the student begins with zero and multiplies each number by five. The first one around the clock wins a point for his team.

Variation: Addition
Division
Subtraction

(To save time--put these clock faces on overhead transparencies and project these on board--write answer directly on board. ed.)



Race to the Top

Materials: A piece of wood (approx. 12 inches by 4 inches), 2 pieces of construction paper (if you have a big piece of construction paper, fold it in half and cut it into two pieces), a pencil, a large piece of thin cardboard (about 34 inches by 34 inches), a ruler, glue or stapler, 2 pegs (approx. 1 inch long by 1/8 inch thick), pair of scissors.

Directions:

Take one piece of the construction paper and locate the center. Now cut a hole in the center of it 3 inches across and 1 inch high. Now do the same thing with the other piece of construction paper, but make sure that the hole is the same place as in the other sheet. Glue these two pieces of paper together along each side (or staple) but not across the bottom or top. Now put this to the side.

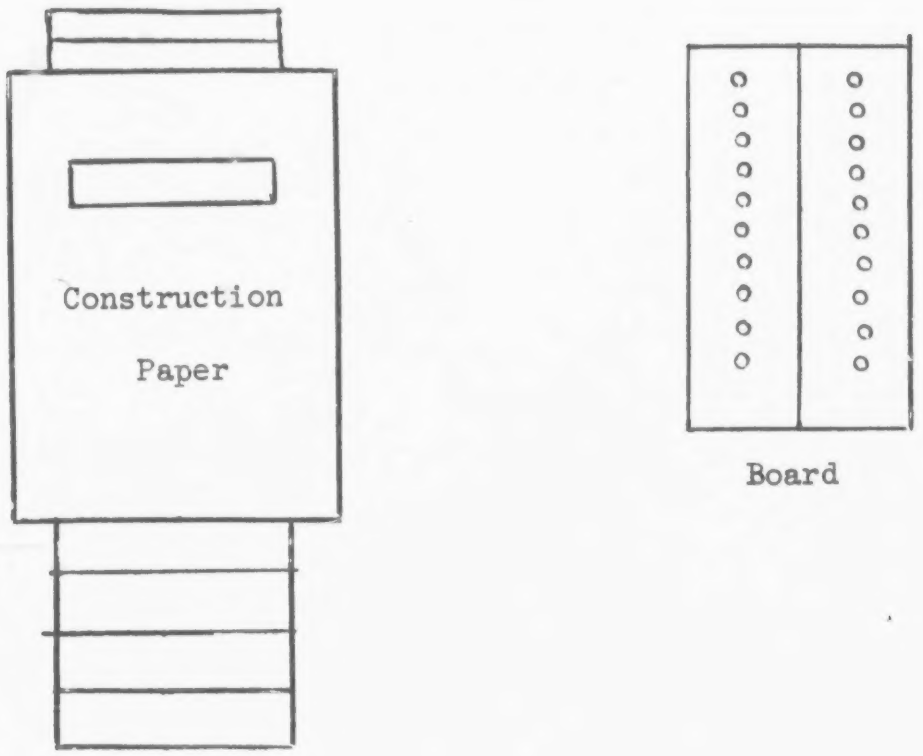
Pick up the piece of wood and drill about 8 holes in a row on each side as shown in the sketch. Make sure that the holes are just big enough for the pegs to fit in. Don't drill the holes all the way through. Put the pegs in the holes so they don't get lost.

Now take the cardboard and cut it into pieces 18 inches long and 6 inches wide. Mark the pieces into 18 boxes on each side of the cardboard and each piece of cardboard. Make up some hard problems and print each of them in a box on one side of the cardboard. Print the answer for that question on the back of that same cardboard right behind where the question is. Now put a piece of the cardboard in between the 2 pieces of construction paper and push in until only one question shows in the box hole in the paper. Try to answer the question and then turn the game over and check for the right answer in the box on the back.

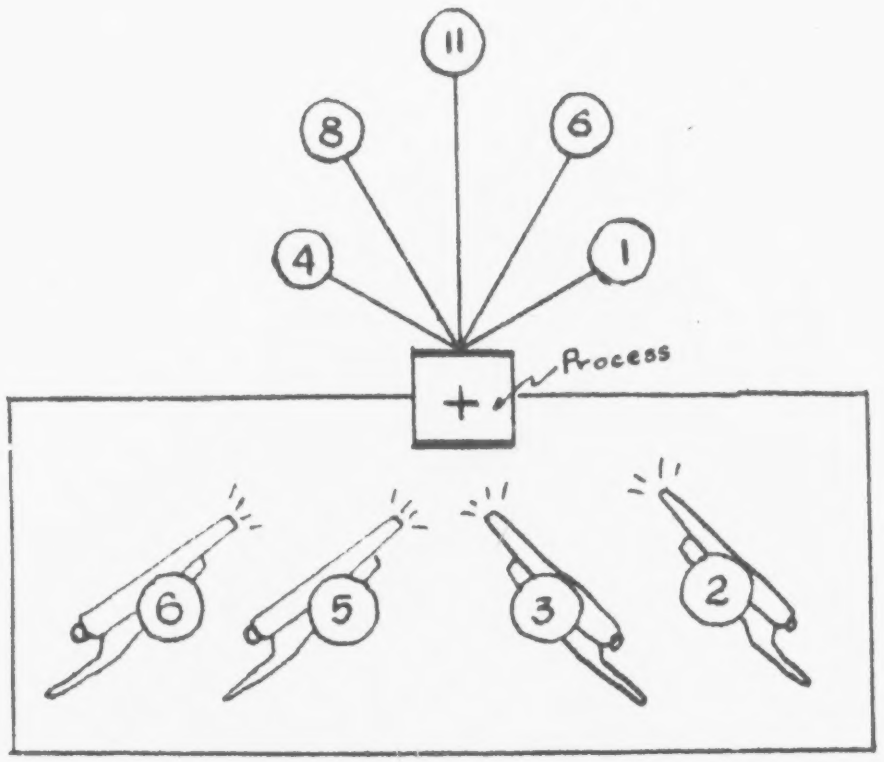
Play this game with one of your friends. You take one row of holes on the board and your friend take the other. Start at the bottom. When you answer a question correctly move your peg up one hole, if it is wrong you can't move. Now it is your friend's turn. RACE TO THE TOP.

(See next page)

Race to the Top (Cont.)



Blast 'Em



(This can be done with an overhead transparency with numerals and process written on the transparency with grease pencil. Then they may be changed simply by wiping them off with towel.)

"BUZZ" - Multiplication Facts Game

Children count, in turn, around the room. When the number fact being studied (example: 7) or one of its multiples (example: 7, 14, 21, 28, etc.) comes up, that child says "buzz" instead. Counting can be "up" or "down" or can be complicated by saying "buzz" for any number with a 7 in it (example: 17, 27, etc.)

Chalkboard Relay Game

Choose two or more teams. Keep teams to no more than eight players. Each team lines up in a single column. At a signal, pupil in front runs to the chalkboard and writes a number. Decide in advance if numbers are to be one, two, or three digits. He returns and hands the chalk to his next team mate who goes to the board and writes a different number under the first. This continues until all team members have written a number. The last one on a team adds the column and becomes number one in the line. The game continues until every child in line has a turn at adding. Give ten points for each correctly added problem.

Number Game

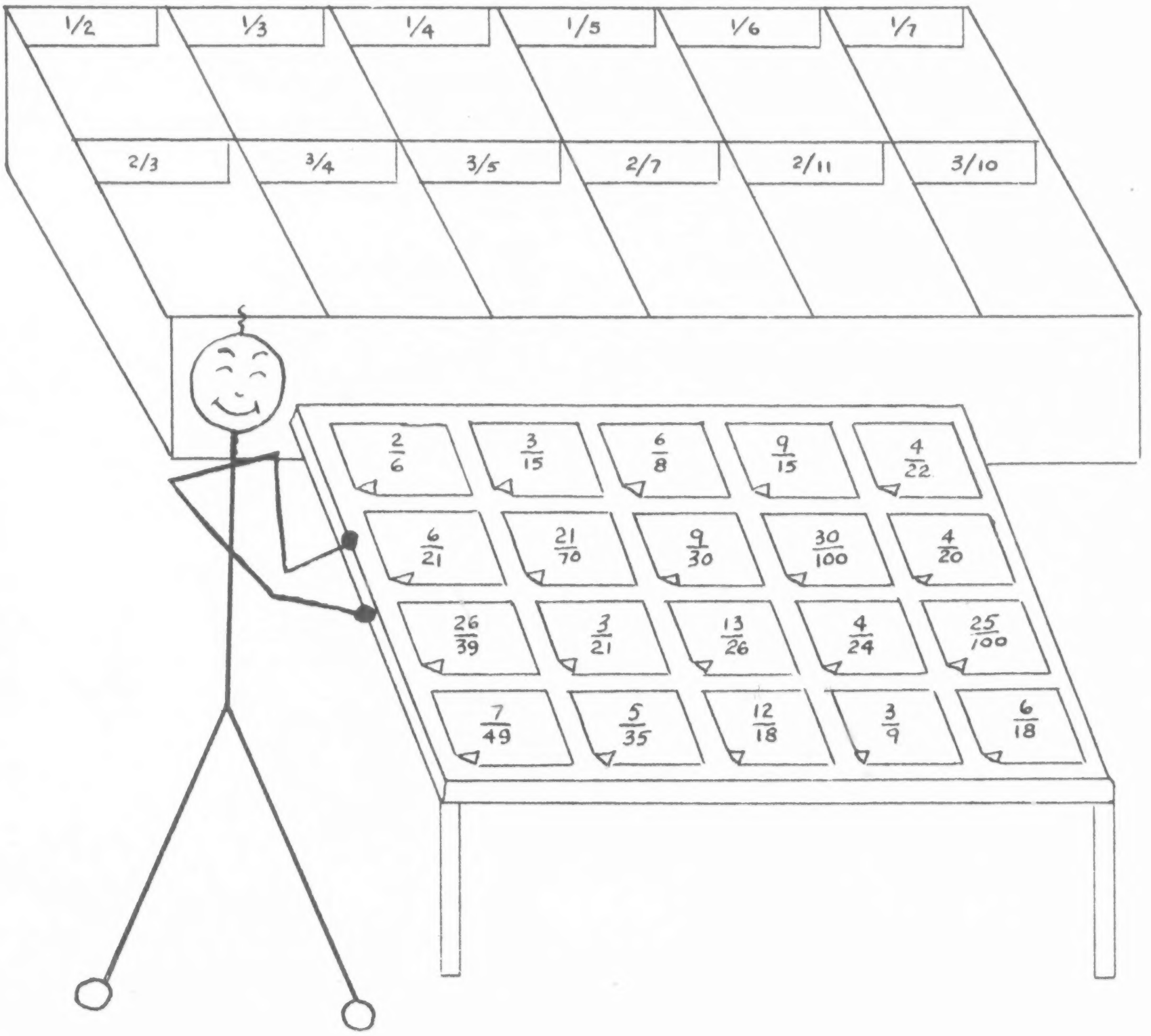
How far can you go using only the digit 4 and any operation???

$0 = 4 - 4$	$8 =$ ____	$16 =$ ____
$1 = 4 \div 4$	$9 =$ ____	$17 =$ ____
$2 = \frac{4}{4} + \frac{4}{4}$	$10 =$ ____	$18 =$ ____
$3 =$ ____	$11 =$ ____	$19 =$ ____
$4 =$ ____	$12 =$ ____	$20 =$ ____
$5 =$ ____	$13 =$ ____	$21 =$ ____
$6 =$ ____	$14 =$ ____	
$7 =$ ____	$15 =$ ____	

Liams

Yelof works in the Liam office. Every day it is his job to sort the liams into bins. The liams are labeled with rational numbers, and Yelof's job is to place each liam into the bin that is the equivalent of the number on theliam. See if you can do Yelof's job.

Example: Put $\frac{2}{6}$ in the bin marked $\frac{1}{3}$. As you know, $\frac{2}{6} = \frac{2 \times 1}{2 \times 3} = \frac{1}{3}$



Team Learning of Geometry in Grades 5 to 8
(They Will Study)

Form two teams -- boys against girls if the balance is about right. Or select the teams as you wish. Letting them choose up sides is rather cruel for the last ones chosen.

Give the teams two or three days to study the geometry unit -- to learn the constructions, relationships, terms, or whatever you want them to learn. Require each student to make up 20 questions on the material.

Then have the teams face each other across the room for the following contest:

Flip a coin to see who kicks off (asks the first question). If your teams are boys and girls, suppose the girls win the toss. A girl asks a question and calls on one of the boys who has his hand up. If the boy answers correctly, his team gets 5 points. If he misses (or no boy's hand went up) the girl may call on a member of her own team who has her hand up. If this girl answers correctly, her team gets 4 points. If she misses, the boys get a second chance to answer for three points. If the boy misses, the girls get another chance - for 2 points. If no one answers correctly, the girls get a point by default. Then a boy asks a question and the same procedure is followed.

The contest should go on for more than one day. Some students will be motivated to study for the second round. The students should be encouraged to develop questions that call for more than just a factual answer -- although the game itself tends to promote this after the first day of play.

Keeping score at the chalkboard provides considerable stimulation, especially when the scores are close. The teacher must referee in such matters as whether an answer is correct, whether a question is "fair", whether the answer is to be found in the material studied.

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